LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2015**

MT 5509 - ALGEBRAIC STRUCTURE - II

Date : 25/09/2015 Dept. No. Max. : 100 Marks

Time : 09:00-12:00

PART A

ANSWERALL THE QUESTIONS $(\mathbf{10 * 2 = 20}\ \boldsymbol{marks})$

1. Define a vector space over a field F.
2. Show that the vectors (1,1) and (−3,2) in R^2 are linearly independent over R ,the field of real numbers.
3. Define a basis of a vector space.
4. Verify that $T : R^2 \to R$ defined by $T(a,b) = ab$ for all $a,b \in R$ is a vector space homomorphism .
5. Let R^3 be the inner product over R under standard inner product. Find the norm of (3,0,4).
6. Define eigen value of linear transformation of T.
7. Define a symmetric matrix and give an example.
8. Define trace of a matrix and give an example.
9. Find the rank of the matrix A= $\begin{pmatrix} 1 & 5 & -7 \\ 2 & 3 & 1 \end{pmatrix}$ over the field of rational numbers.
10. If $T \in A(V)$ and then prove that .

PART B

ANSWERANY FIVE QUESTIONS $(\mathbf{5 * 8 = 40}\ \boldsymbol{marks})$

11. If V is a vector space over F then show that

i) for

ii) for .

iii) If , then implies that .

12. Show that a nonempty subset W of a vector space V over F is a subspace of V if and only if $aw_{1_1} + bw_2 \in W$ *for all* $a, b \in F, w_1, w_2 \in W$.

13. Let V be a vector space and suppose that one basis has n elements and another basis has m elements .Then prove that $m = n$.

14. If V and W are two n-dimensional vector spaces over F. Then prove that any isomorphism $T$of V onto W maps a basis of V onto a basis of W.

15. State and prove Schwarz inequality.

16. Show that any square matrix A can be expressed uniquely as the sum of a symmetric matrix and a skew-symmetric matrix.

17. For what values of λ the system of equations $x_1 + x_2 + x_3 = 1, x_1 + 2x_2 + 4x_3 = \lambda, x_1 + 4x_2 + 10x_3 = \lambda^2$ over the rational field is consistent ?

18. a) If $T \in A(V)$ is skew-Hermitian, prove that all of its eigenvalues are pure imaginaries.

b)Prove that the eigenvalues of a unitary transformation are all of absolute value 1.

PART C

ANSWER ANY TWO QUESTIONS $(\mathbf{2 * 20 = 40}\ \boldsymbol{marks})$

19. a) Prove that the vector space V over F is a direct sum of two of its subspaces W_1 and W_2 if and only if $V = W_1 + W_2$ and $W_1 \cap W_2 = \{0\}$.

1. If S and T are subsets of a vector space V over F,then prove that

$$L(S \cup T) = L(S) + L(T).$$

2. If W_1 and W_1 are subspaces of a finite dimensional vector space V, prove that $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

3. State and prove Gram-Schmidt orthonormalization process.

4. a) Let V be a vector space of dimension n over F, and let $T \in A(V)$. If $m_1(T)$ and $m_2(T)$ are the matrices of T relative to two bases $\{v_1, \dots v_n\}$ and $\{w_1, \dots w_n\}$ of V, respectively. Then prove that there is an invertible matrix C in F_n such that $m_2(T) = Cm_1(T)C^{-1}$.

b) Prove that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V onto an orthonormal basis of V.

****